C–7–C

Roll No. ....................................

Total No. of Questions : 31] [Total No. of Printed Pages : 8

# $12^{th}$ARA(SZ)JK/LEHUT2024
# 1507-C
# MATHEMATICS

**Time : 3 Hours]** **[Maximum Marks : 80**

***General Instructions*** **:**

1. This question paper contains four sections A, B, C and D. Each Section is compulsory.
2. Section A—Question 1 to 10 comprises of Very Very Short Answer Type Questions of 1 mark each.
3. Section B—Question 11 to 20 comprises of 10 Very Short Answer (VSA) Type Questions of 2 marks each.
4. Section C—Question 21 to 28 comprises of 8 Short Answer (SA) Type Questions of 4 marks each.
5. Section D—Question 29 to 31 comprises of Long Answer (LA) Type Questions of 6 marks each.

## SECTION–A

**(VERY VERY SHORT ANSWER TYPE QUESTIONS)** 1 each

**1.** If

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

then $x$ is equal to :

(A) $-2$ (B) 2

(C) 3 (D) 4

**2.** If

$$A = \begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$$

and $A + A' = I$

then the value of $\alpha$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\pi$ (D) $\frac{3\pi}{2}$

**3.** The number of arbitrary constants in the general solution of a differential equation of 4th order is :

(A) 0 (B) 2

(C) 3 (D) 4

**4.** Second order derivative of $e^x \sin 5x$ is .................. .

**5.** $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to :

(A) $\tan x + \cot x + C$ (B) $\tan x + \operatorname{cosec} x + C$

(C) $-\tan x + \cot x + C$ (D) $\tan x + \sec x + C$

**6.** The rate of change of the area of a circle with respect to its radius $r$ when $r = 4$ is .................. .

**7.** $\int \frac{e^{x}(1+x)}{\cos^{2}(x.e^{x})}dx$ is equal to :

(A) $-\cot(e^{x}.x) + C$ (B) $\tan(e^{x}.x) + C$

(C) $\tan(e^{x}) + C$ (D) $\cot(e^{x}) + C$

**8.** Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3$, $|\vec{b}|=\frac{\sqrt{2}}{3}$, then $\vec{a}\times\vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{3}$

**9.** The vector equation of XZ plane is :

(A) $\vec{r}\cdot\hat{k}=0$ (B) $\vec{r}\cdot\hat{j}=0$

(C) $\vec{r}\cdot\hat{i}=0$ (D) $\vec{r}\cdot\vec{d}=0$

**10.** Define feasible region.

## SECTION–B

**(VERY SHORT ANSWER TYPE QUESTIONS)** 2 each

**11.** Determine the relation in the set Z of all integers defined as R = $\{(x, y) \mid x - y$ is an integer$\}$ is reflexive, symmetric and transitive.

**12.** Find the principal value of

$$\operatorname{cosec}^{-1}(2)$$

**13.** Find the interval in which the function $6 - 9x - x^2$ is strictly increasing or decreasing.

**14.** For given vectors :

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

and
$$\vec{b} = -\hat{i} + \hat{j} - \hat{k}$$

find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.

**15.** Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

**16.** Evaluate :

$$\int \frac{1 - \cos x}{1 + \cos x} dx$$

**17.** Using the properties of definite integrals, evaluate :

$$\int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx$$

**18.** Compute $P\left(\frac{A}{B}\right)$, if P(B) = 0.5 and P(A ∩ B) = 0.32.

**19.** If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$. Find :

(i) $P(A \cap B)$

(ii) $P\left(\frac{A}{B}\right)$

**20.** Construct a $2 \times 2$ matrix, $A = [a_{ij}]$, whose elements are given by :

$$a_{ij} = \frac{(i+2j)^2}{2}$$

## SECTION–C

### (SHORT ANSWER TYPE QUESTIONS) 4 each

**21.** Solve the differential equation :

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

**22.** Evaluate using the properties of definite integrals :

$$\int_0^{\pi/4} \log(1 + \tan x) dx$$

**23.** If

$$x = \sqrt{a^{\sin^{-1} t}}, \, y = \sqrt{a^{\cos^{-1} t}}$$

show that :

$$\frac{dy}{dx} = -\frac{y}{x}$$

24. Find the shortest distance between the lines :

$$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$$

and

$$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}.$$

25. Show that the vectors

$$2\hat{i}-\hat{j}+\hat{k},\ \hat{i}-3\hat{j}-5\hat{k}$$

and

$$3\hat{i}-4\hat{j}-4\hat{k}$$

form the vertices of a right-angled triangle.

26. Solve the following linear programming problem graphically :

**Maximize :** 

$$Z = 5x + 3y$$

**subject to :**

$$3x + 5y \leq 15$$

$$5x + 2y \leq 10$$

$$x \geq 0,\ y \geq 0$$

**27.** Let $f : \mathrm{N} \rightarrow \mathrm{N}$ be defined by :

$$f(n)=\begin{cases}\dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even}\end{cases} \quad \forall n \in \mathrm{N}$$

State whether the function is bijective, justify your answer.

**28.** A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag.

## SECTION–D

## (LONG ANSWER TYPE QUESTIONS) 6 each

**29.** Solve the following system of equation by matrix method :

$$2x + y + z = 1,$$

$$x - 2y - z = \frac{3}{2},$$

$$3y - 5z = 9$$

*Or*

Express $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$ as the sum of symmetric and skew-symmetric matrix.

**30.** Evaluate the integral

$$\int \frac{2x-3}{(x^2-1)(2x+3)}dx$$

*Or*

Evaluate

$$\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$$

**31.** If $y = (\tan^{-1} x)^2$, then show that :

$$(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$$

*Or*

Differentiate w.r.t. $x$ :

$$x^{x\cos x} + \frac{x^2+1}{x^2-1}.$$