

B-7-X

Roll No.

Total No. of Questions : 31]

[Total No. of Printed Pages : 8

12thARF(SZ)JKUT2024-25
307-X
MATHEMATICS

Time : 3 Hours]

[Maximum Marks : 80

SECTION-A
(OBJECTIVE TYPE QUESTIONS/
MULTIPLE CHOICE QUESTIONS)

1 each

1. If

$$\begin{vmatrix} x & 3 \\ 4 & -2 \end{vmatrix} = 0$$

then x is equal to :

- (A) 6
 - (B) -6
 - (C) 4
 - (D) -4
2. For a square matrix A , $A(\text{adj } A) = \dots\dots\dots$

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Turn Over

3. Derivative of $e^{\log \tan x}$ w.r.t. x is :

- (A) $\cot x$
- (B) $\tan x$
- (C) $\sec^2 x$
- (D) $\operatorname{cosec}^2 x$

4. Second derivative of $\log (\log x)$ w.r.t. x is :

- (A) $\frac{1}{x \log x}$
- (B) $\frac{1}{\log x}$
- (C) $\frac{1}{x}$
- (D) $\frac{-(1 + \log x)}{(x \log x)^2}$

5. The function :

$$f(x) = \frac{|x|}{x}, \quad x \neq 0$$

$$= 0, \quad x = 0$$

is discontinuous at $x = 0$.

(True/False)

6. Maximum value of $\sin x$ is :

- (A) 1
- (B) -1
- (C) 2
- (D) -2

(3)

7. $\int \tan x \, dx$ is equal to :

(A) $\log \cos x + C$

(B) $\log \sec x + C$

(C) $\log \sin x + C$

(D) $\log \operatorname{cosec} x + C$

8. Two lines are parallel if their direction ratios are

9. Projection of $\hat{i}$ on $\hat{j}$ is zero. (True/False)

10. Define optimal solution.

SECTION-B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

11. Show that the relation R in the set $\{1, 2, 3\}$ given by

$$R = \{(1, 2), (2, 1)\}$$

is symmetric but neither reflexive nor transitive.

12. Prove that :

$$3 \sin^{-1} x = \sin^{-1} (3x - 4x^3)$$

$$x \in \left[-\frac{1}{2}, \frac{1}{2} \right]$$

13. The length 'x' of a rectangle is decreasing at the rate of 3 cm/minute and the width 'y' is increasing at the rate of 2 cm/minute. When $x = 10$ cm and $y = 6$ cm, find the rate of change of the perimeter.

14. Find the unit vector in the direction of the vector

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$

15. Find $|\vec{x}|$, if for a unit vector $\vec{a}$,

$$(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 12$$

16. Find :

$$\int \frac{\sin x}{1 + \cos x} dx$$

17. Evaluate :

$$\int_2^3 \frac{x dx}{x^2 + 1}$$

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(5)

18. A coin is tossed three times. If E is the event 'head on third toss' and F is the event 'heads on first two tosses'. Find $P(E | F)$.
19. Let E and F be two events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent ?
20. Find the product :

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

SECTION-C

(SHORT ANSWER TYPE QUESTIONS)

4 each

21. Find the relationship between a and b so that the function f defined by <https://www.jkboseonline.com>

$$\begin{aligned} f(x) &= ax + 1 \text{ if } x \leq 3 \\ &= bx + 3 \text{ if } x > 3 \end{aligned}$$

is continuous at $x = 3$.

22. Evaluate :

$$\int_0^1 x(1-x)^n dx$$

23. Differentiate $(\log x)^x + x^{\log x}$ w.r.t. x .

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Turn Over

24. Find the shortest distance between the lines :

$$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$$

and

$$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$$

25. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that :

$$\vec{a} + \vec{b} + \vec{c} = 0$$

find the value of

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

26. Solve the linear programming problem graphically :

$$\text{Maximize } z = 3x + 2y$$

Subject to :

$$x + 2y \leq 10,$$

$$3x + y \leq 15,$$

$$x, y \geq 0.$$

27. Let A and B be two sets. Show that :

$$f : A \times B \rightarrow B \times A$$

such that $f(a, b) = (b, a)$ is bijective function.

28. An urn contains 5 red and 5 black balls. A ball is drawn at random, its colour is noted and is returned to the urn. Moreover, 2 additional balls of the colour drawn are put in the urn and then a ball is drawn at random. What is the probability that the second ball is red ?

SECTION-D

(LONG ANSWER TYPE QUESTIONS)

6 each

29. For a matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

show $A^3 - 6A^2 + 5A + 11I = 0$.

Or

Solve the system of linear equations, using matrix method :

$$2x + 3y + 3z = 5$$

$$x - 2y + z = -4$$

$$3x - y - 2z = 3$$

30. Evaluate :

$$\int_0^{\pi/2} \sqrt{\sin \phi} \cos^5 \phi \, d\phi$$

Or

Evaluate :

$$\int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} \, dx$$

31. Find the local maxima and local minima, if any, of the function :

$$f(x) = \sin x - \cos x, \quad 0 < x < 2\pi.$$

Or

Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.