**B–5–C**

Roll No.....

Total No. of Questions : **29** ] [ Total No. of Printed Pages : **8**

# *XIIKDAR21*

# *5005–C*

# *MATHEMATICS*

**Time : 3 Hours ]** **[ Maximum Marks : 100**

## Section–A

**(Multiple Choice Questions)** 1 each

1. Let R be the relation in the set N given by $R = \{(a, b) \mid a = b - 2, (b > 6)\}$ :

   (A) $(2, 4) \in R$ (B) $(3, 8) \in R$

   (C) $(6, 8) \in R$ (D) $(8, 7) \in R$

   (Choose the correct answer)

2. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to :

   (A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$

   (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$ (Choose the correct answer)

3. A and B are symmetric matrices of same order, then AB – BA is a :

(A) Skew symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix (Choose the correct answer)

4. The value of :

$$\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{k}\times\hat{i})+\hat{k}\cdot(\hat{j}\times\hat{i})$$

is :

(A) 0 (B) –1

(C) 1 (D) 3 (Choose the correct answer)

## Section–B

## (Very Short Answer Type Questions)

5. Evaluate :

$$\int\frac{\sin(\tan^{-1}x)dx}{1+x^2}$$

6. Evaluate :

$$\int_0^1\frac{dx}{\sqrt{1-x^2}}$$

7. Find the values of $x$, $y$ and $z$ from the equation :

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

8. Find the order and degree of differential equation :

$$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 4$$

9. If a line has direction ratios 2, –1, 2, determine its direction cosines.

10. Define objective function and optimal solution of L.P.P.

11. $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$, $P(A \cup B) = \frac{7}{11}$, find $P(A \cap B)$.

12. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $(A \cap B)$ if A and B are independent events.

## Section–C

**(Short Answer Type Questions)** 4 each

13. If :

$$f(x) = \frac{4x+3}{6x-4} \qquad x \neq \frac{2}{3}.$$

Show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. What is the inverse of $f$ ?

14. Write in the simplest form :

$$\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$$

15. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & 3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

16. Find the equation of tangent and normal to the curve at the indicated point $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(0, 5)$.

17. Use differentials, find the approximate value up to 3 places of decimal $(25)^{1/3}$.

18. Integrate the function $x \tan^{-1} x$.

19. Find the relationship between $a$ and $b$ such that the function $f$ defined by :

$$f(x) = \begin{cases} ax + 1 & x \leq 3 \\ bx + 3 & x > 3 \end{cases}$$

is continuous at $x = 3$.

20. If $y = \sin^{-1} x$, show that :

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

21. Find the unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

22. Find the angle between two planes :

$$2x + y - 2z = 5$$

and

$$3x - 6y - 2z = 7$$

using vector method.

23. Solve graphically (L.P.P.) :

Maximize :

$$Z = 4x + y$$

Subject to the constraints :

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0,\ y \geq 0$$

## Section–D

**(Long Answer Type Questions)** 6 each

24. Using the properties of determinants, Prove that :

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

*Or*

Solve the system of equations by matrix method :

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

25. If $y = x^{\sin x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$.

*Or*

Show that of all the rectangles inscribed in a given fixed circle the square has the maximum area.

26. Find :

$$\int \frac{(3\sin x - 2)\cos x \, dx}{5 - \cos^2 x - 4\sin x}$$

*Or*

Find the area of the region bounded by the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

27. Find the general solution of the differential equation given by :

$$x\frac{dy}{dx} + 2y = x^2 \log x$$

*Or*

**Find the general solution of the differential equation :**

$$\frac{dy}{dx} = \frac{x+1}{2-y}, \quad y \neq 2$$

28. **Find the equation of the plane passing through three points** (1, 1, 0) (1, 2, 1), (–2, 2, –1).

*Or*

**Find the shortest distance between the lines whose vector equations are :**

$$\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$$

**and**

$$\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

29. Find the probability distribution of the number of doublets in three throws of a pair of dice.

*Or*

A die is thrown 6 times if "getting an odd number" is a success. What is the probability of :

(i) 5 success

(ii) at least 5 success

(iii) at most 5 success