A–6–A

Roll No. .....

Total No. of Questions : **29**]

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12thSZARJD22

6006–A

MATHEMATICS

Time : 2.30 Hours]

[Maximum Marks : 100

Section–A

(Multiple Choice Questions) 1 each

1. The relation R in the set {1, 2, 3} given by R = {(x, y) | x < y, x, y ∈ A} is :

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) Anti-symmetric

2. The principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is :

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{3\pi}{4}$

3. Two matrices A and B of the same order are said to be equal, if :

(A) $a_{ij} = 0$ (B) $b_{ij} = 0$

(C) $a_{ij} + b_{ij} = 0$ (D) $a_{ij} = b_{ij}$ for all i, j

4. The direction cosines of a unit vector along x-axis are :

(A) (1, 0, 0) (B) (0, 1, 0)

(C) (0, 0, 1) (D) (1, 1, 1)

Section–B

(Very Short Answer Type Questions) 2 each

5. Find the values of x, y and z from the following equation :

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

6. Evaluate :

$$\int (ax^2 + bx + c)dx$$

7. Find the rate of change of area of a circle with respect to its radius when $r = 5$ cm.

8. Form the differential equation representing the family of curve $y = mx^2$, where m is arbitrary constant.

9. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

10. Solve the following L.P.P. graphically :

Maximise :

$$Z = 3x + 4y$$

Subject to constraints :

$$x + y \leq 4,$$

$$x \geq 0, \; y \geq 0$$

11. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

12. If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.

Section–C

(Short Answer Type Questions) 4 each

13. Show that $f : (-1\ \ 1) \to R$ given by $f(x) = \dfrac{x}{x+2}$ is one-one. Find the inverse of the function $f : (-1\ \ 1) \to$ Range of f.

14. Solve :

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

15. For the matrix A and B verify that $(AB)' = B'A'$, $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$.

16. Find the relationship between a and b so that the function f defined by :

$$f(x) = \begin{cases} ax + 1 & \text{if} \quad x \leq 3 \\ bx + 3 & \text{if} \quad x > 3 \end{cases}$$

is continuous at $x = 3$.

17. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is :

(a) Strictly increasing

(b) Strictly decreasing

18. Find the equations of the tangent and normal to the given curve at the indicated point $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5).

19. Find the general solution of the differential equation :

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

20. Find the area of the region bounded by the ellipse :

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

21. Find $|\vec{a} \times \vec{b}|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

22. If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

23. Solve the following problem graphically :

Minimise and maximise : $Z = 3x + 9y$

Subject to the linear constraints :

$$x + 3y \leq 60,\ x \leq y,$$

and

$$x + y \geq 10,\ x \geq 0,\ y \geq 0$$

Section–D

(Long Answer Type Questions)

24. By using properties of determinants prove that :

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

Or

If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then verify that A(adj A) = | A |I. Also find A^{-1}.

25. Find $\frac{dy}{dx}$ of the function $x^y + y^x = 1$.

Or

If $y = 3 \cos (\log x) + 4 \sin (\log x)$, show that :

$$x^2y_2 + xy_1 + y = 0$$

26. Integrate the rational fraction :

$$\frac{2x-3}{(x^2-1)(2x+3)}$$

(/)

Or

Using the properties of definite integrals evaluate :

$$\int_{-5}^{5} |x+2|\,dx$$

27. Find the general solution of the differential equation :

$$x\frac{dy}{dx}+2y=x^2\log x$$

Or

Show that the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is homogeneous and solve it.

28. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2, 2, 1)$.

Or

Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

29. Find the probability distribution of number of doublets in three throws of a pair of dice.

Or

Two balls are drawn at random with replacement from a box containing 10 black balls and 8 red balls. Find the probability that :

(a) Both balls are red

(b) First ball is black and second is red

(c) One of them is black and other is red