

B-12-X

Roll No.

Total No. of Questions : 29]

[Total No. of Printed Pages : 8

XIIARJKUT23
9112-X
MATHEMATICS

[Maximum Marks : 100

Time : 3 Hours]

SECTION-A

(MULTIPLE CHOICE QUESTIONS)

1 each

1. Range of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined as $f(x) = x^2$ is :

(A) $(0, \infty)$

(B) $(-\infty, 0)$

(C) $[0, \infty)$

(D) $(-\infty, 0]$

2. $\sin^{-1} x + \cos^{-1} x, x \in [-1, 1]$ is equal to :

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) π

3. If A is a square matrix of order n , then $A(\text{adj } A) = \dots\dots\dots$

(A) $|A|$

(B) I

(C) $|A| I$

(D) None of these

4. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then $\vec{a} \cdot \vec{b} = \dots\dots\dots$

(A) $\cos \theta$

(B) $\sin \theta$

(C) $ab \cos \theta$

(D) $ab \sin \theta$

SECTION-B

(VERY SHORT ANSWER TYPE QUESTIONS) 2 each

5. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, find the values of x .

6. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

7. Differentiate $\sin(x^2 + 5)$ with respect to x .

8. Find :

$$\int \frac{x^3 + 5x^2 - 4}{x^2} dx$$

9. A coin is tossed three times. Find $P(E/F)$, where E is the event "head on third toss" and F is the event "heads on first two tosses".

10. Compute $P(A \cap B)$, where $P(A) = 0.8$, $P(B) = 0.5$ and $P(B/A) = 0.4$.

11. Find the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ and having magnitude of 8 units.

12. Define Linear Constraints.

SECTION-C

(SHORT ANSWER TYPE QUESTIONS)

4 each

✓ 13. Find $g \circ f$ and $f \circ g$ if $f(x) = |x|$ and $g(x) = |5x - 2|$.

✓ 14. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, find the value of x .

✓ 15. If :

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

prove that $A^3 - 6A^2 + 7A + 2I = 0$.

✓ 16. Find local maxima and local minima if any of the function :

$$f(x) = x^3 - 6x^2 + 9x + 15$$

✓ 17. Find general solution of differential equation :

$$x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$$

✓ 18. Find $\frac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$.

~~19.~~ Prove that :

$$\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

20. If $y = (\tan^{-1} x)^2$, show that :

$$(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$$

~~21.~~ If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

22. Find the Vector and Cartesian equations of the line that passes through the points (3, -2, -5) and (3, -2, 6).

~~23.~~ Solve the following graphically :

Minimise :

$$Z = x + 2y$$

Subject to the constraints :

$$2x + y \geq 3$$

$$x + 2y \geq 6$$

$$x, y \geq 0$$

SECTION-D**(LONG ANSWER TYPE QUESTIONS)**

6 each

24. Using properties of determinants show that :

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$$

Or

Solve the system of linear equations using matrix method :

$$2x + y + z = 1$$

$$x - 2y - z = \frac{3}{2}$$

$$3y - 5z = 9$$

25. Find $\frac{dy}{dx}$ if $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$.

Or

Find $\frac{dy}{dx}$ if $(\cos x)^y = (\cos y)^x$.

26. Find $\int_0^{\pi/2} \sqrt{\sin \phi} \cos^5 \phi \, d\phi$.

Or

Using integration, find the area of region bounded by triangle whose vertices are $(-1, 0)$, $(1, 3)$, $(3, 2)$.

27. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, find the vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 15$.

Or

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad \text{https://www.jkboseonline.com}$$

28. Find the intervals in which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or decreasing.

Or

Evaluate :

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} \, dx$$

(8)

29. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item.

Or

Find the mean and variance of the number obtained on a throw of an unbiased die.